Released
Advanced Mathematics Items
Population 3
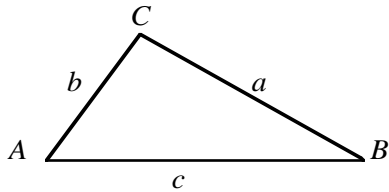
IEA

TIMSS

MATHEMATICS NOTATION

Vector: $\overrightarrow{r}$ or $\overrightarrow{AB}$ Magnitude of vector: r or $|\overrightarrow{r}|$

SELECTED MATHEMATICS FORMULAE

Triangles

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\sin(A + B) = \sin A \, \cos B + \cos A \, \sin B$$

$$\cos(A + B) = \cos A \, \cos B - \sin A \, \sin B$$

Logarithms

If $a>0$, $b>0$ and $b \neq 1$, $c>0$ and $c \neq 1$

$$\log_b a = \frac{\log_c a}{\log_c b}$$

Sequences

If t_n is the general term of the arithmetic sequence with first term a and with constant difference d, then:

$$t_n = a + (n - 1)d$$

If S_n is the sum of the first n consecutive terms of an arithmetic sequence with first term t_1, then:

$$S_n = \frac{n(t_1 + t_n)}{2}$$

If t_n is the general term of the geometrical sequence with first term a and with constant ratio r, then $t_n = ar^{n-1}$

If S_n is the sum of the first n consecutive terms of a geometrical sequence with first term a and with constant ratio r, where $-1 < r < 1$, then:

$$\lim_{n \to +\infty} S_n = \frac{a}{1 - r}$$

If $z = x + iy = r(\cos A + i\sin A)$, $(x, y) \in R^2$ then: $z^n = [r(\cos A + i\sin A)]^n$
$$= r^n(\cos nA + i\sin nA)$$

(Continued on the next page.)

SELECTED MATHEMATICS FORMULAE
(Continued)

<u>Length, Area, and Volume.</u>

If d is the distance between (x_1, y_1) and (x_2, y_2),

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$A_{\text{cylinder(curved surface)}} = 2\pi rh$

$V_{\text{cylinder}} = \pi r^2 h$

$$V_{\text{cone}} = \frac{\pi r^2 h}{3}$$

<u>Probability</u>

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If $B \neq \emptyset$, $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

If A and B are independent, then

$$P(A \cap B) = P(A)P(B)$$

K1. If $xy = 1$ and x is greater than 0, which of the following statements is true?

    A.   When x is greater than 1, y is negative.

    B.   When x is greater than 1, y is greater than 1.

    C.   When x is less than 1, y is less than 1.

    D.   As x increases, y increases.

    E.   As x increases, y decreases.

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	E	Numbers, Equations and Functions	Complex Procedures	85%	353

K2. In how many ways can one arrange on a bookshelf 5 thick books, 4 medium sized books and 3 thin books so that the books of the same size remain together?

A. $5! \, 4! \, 3! \, 3! = 103\ 680$

B. $5! \, 4! \, 3! = 17\ 280$

C. $(5! \, 4! \, 3!) \times 3 = 51\ 840$

D. $5 \times 4 \times 3 \times 3 = 180$
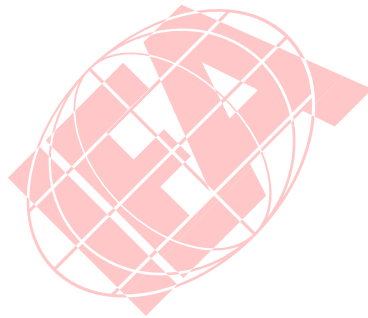
E. $2^{12} \times 3 = 12\ 288$

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	A	Numbers, Equations and Functions	Solving Problems	27%	703

K3. The acceleration of an object moving in a straight line can be determined from

A.   the slope of the distance-time graph

B.   the area below the distance-time graph

C.   the slope of the velocity-time graph
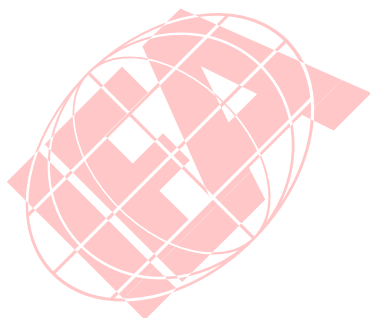
D.   the area below the velocity-time graph

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	C	Calculus	Knowing	65%	489

K4.   The value of $\displaystyle\lim_{h\to 0}\frac{\sqrt{2+h}-\sqrt{2}}{h}$ is

A.   0

B. $\dfrac{1}{2\sqrt{2}}$
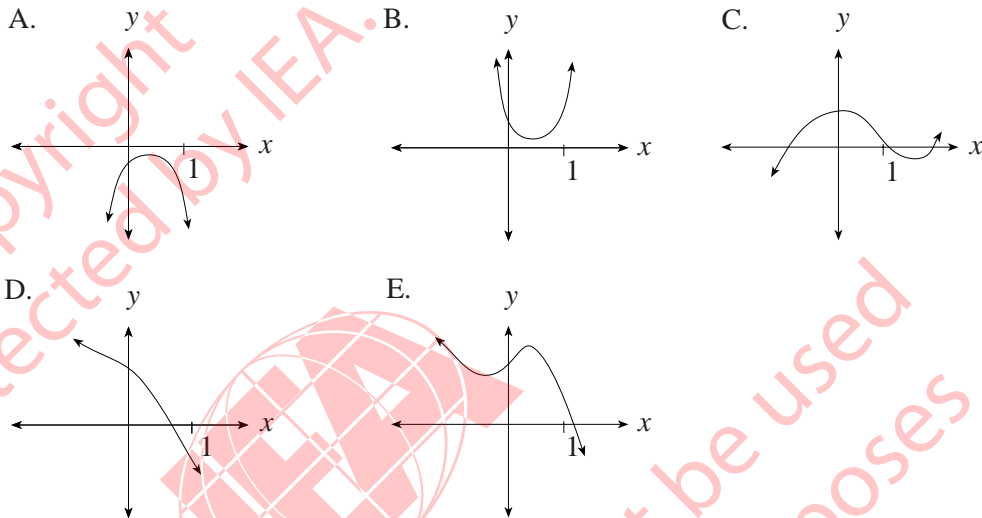
C. $\dfrac{1}{2}$

D. $\dfrac{1}{\sqrt{2}}$

E. ∞

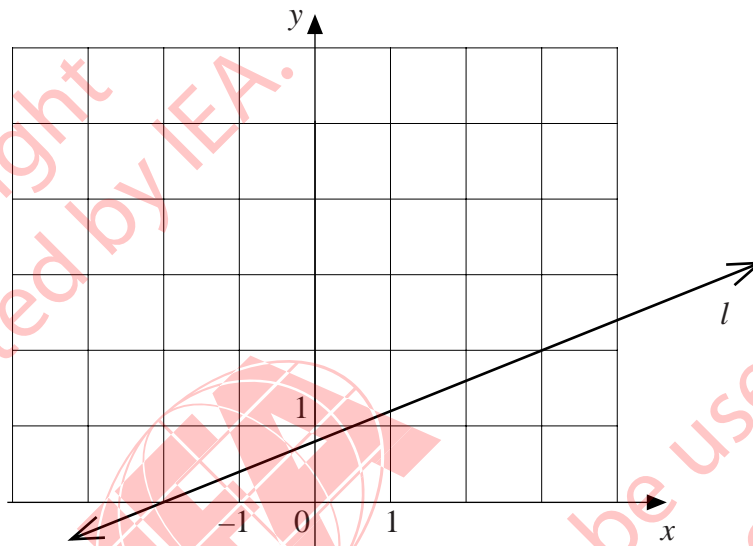| Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	B	Calculus	Routine Procedures	29%	692

K5. Which of the following graphs has these features:
$f'(0) > 0$, $f'(1) < 0$, and $f''(x)$ is always negative?

A.

B.

C.

D.

E.

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	A	Calculus	Solving Problems	45%	601

K6. The line *l* in the figure is the graph of $y = f(x)$.

$\int_{-2}^{3} f(x)\, dx$ is equal to

A. 3

B. 4

C. 4.5

D. 5

E. 5.5

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	D	Calculus	Routine Procedures	58%	537

K7. The vertices of the triangle PQR are the points P(1, 2), Q(4, 6) and R(− 4, 12). Which one of the following statements about triangle PQR is true?

A. PQR is a right triangle with the right angle ∠P.

B. PQR is a right triangle with the right angle ∠Q.

C. PQR is a right triangle with the right angle ∠R.

D. PQR is not a right triangle.

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	B	Geometry	Routine Procedures	56%	547

K8. Which one of the following conics is represented by the equation $(x - 3y)(x + 3y) = 36$?

    A.   Circle

    B.   Ellipse

    C.   Parabola

    D.   Hyperbola

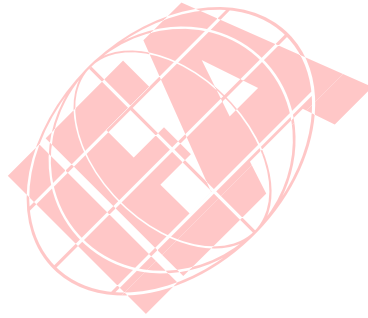| Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	D	Geometry	Knowing	28%	690

K9. Determine the distance between the x-intercept and z-intercept of the plane whose equation is $3x + 2y - 4z = 12$.
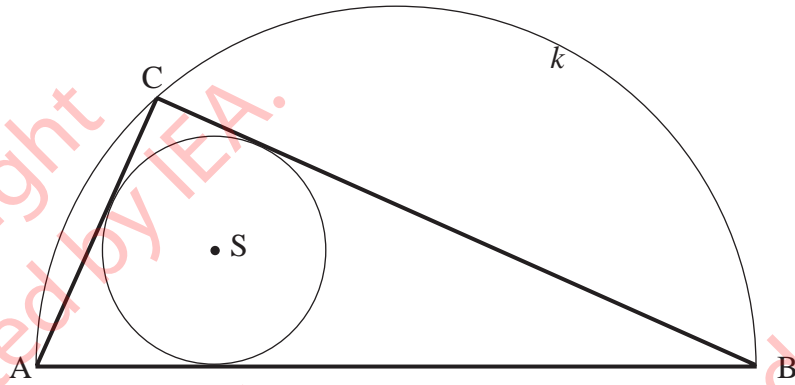
A. $\sqrt{7}$

B. 1

C. 5

D. 7

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	C	Geometry	Routine Procedures	43%	613

K10.

AB is the diameter of a semicircle k, C is an arbitrary point on the semicircle (other than A or B), and S is the centre of the circle inscribed into $\triangle ABC$.

Then the measure of

A. $\angle ASB$ changes as C moves on k.

B. $\angle ASB$ is the same for all positions of C but it cannot be determined without knowing the radius.

C. $\angle ASB = 135°$ for all C.

D. $\angle ASB = 150°$ for all C.

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	C	Geometry	Solving Problems	21%	741

K11. A set of 24 cards is numbered with the positive integers from 1 to 24. If the cards are shuffled and if only one is selected at random, what is the probability that the number on the card is divisible by 4 or 6?

A. $\dfrac{1}{6}$

B. $\dfrac{5}{24}$

C. $\dfrac{1}{4}$

D. $\dfrac{1}{3}$

E. $\dfrac{5}{12}$
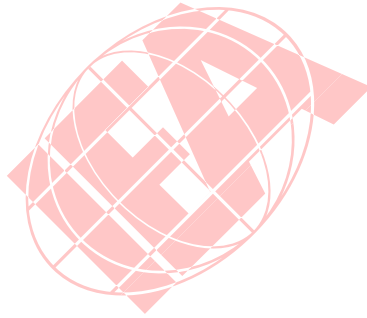
Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	D	Probability & Statistics	Routine Procedures	50%	578

K12. A translation maps A (2,–3) onto A' (–3,–5). Under the same translation, find the coordinates of B' , the image of B (1,4).
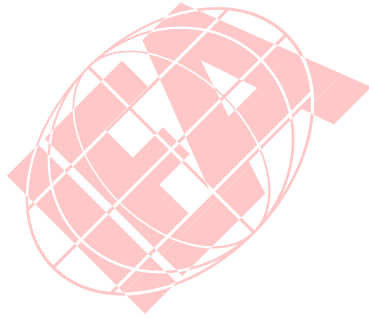
Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	next page	Geometry	Routine Procedures	52%	570

K-12 Coding Guide

K12. A translation maps *A* (2,–3) onto *A′* (–3,–5). Under the same translation, find the coordinates of *B′* , the image of *B* (1,4).

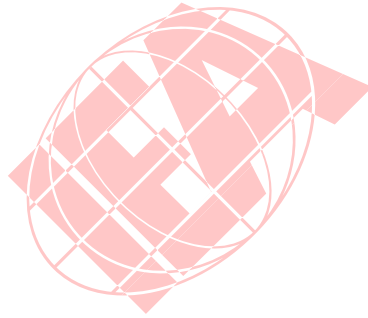| Code	Response
Correct Response	
10	(-4, 2). No work shown, or only points are shown in a diagram such that method cannot be determined.
11	(-4, 2). Method: A diagram that shows more than points is drawn showing the geometrical method used such as mid-point, slope, or change in x- and y-direction.
12	(-4, 2). Method: The coordinates of the translation vector are (-5, -2); the translation vector (-5, -2) is added to B (1,4) to obtain B' (-4, 2). **Note:** If diagram is shown and the translation vector is indicated, also use code 12.
19	Other correct responses with method are shown.
Incorrect Response	
70	Response incorrect. No work shown.
71	(6, 6). Method as in code 12 but uses incorrect translation vector, (5, 2).
72	Method as in code 12 with correct translation vectors (-5, -2) but with error in subtraction of negative numbers.
73	Method as in code 11 with an understandable diagram consisting of more than just points. At least one coordinate of B' is incorrect.
79	Other incorrect responses with method shown. (If no method/work shown, code 70.)
Nonresponse	
90	Crossed-out, illegible, or impossible to interpret.
99	BLANK

K13. The number of bacteria in a colony was growing exponentially. At 1 pm yesterday the number of bacteria was 1000 and at 3 pm yesterday it was 4000.

How many bacteria were there in the colony at 6 pm yesterday?

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	next page	Numbers, Equations and Functions	Solving Problems	27%	710

K-13 Coding Guide

Code	Response
Correct Response	
10	32 000. No work shown.
11	32 000. States explicitly that the number of bacteria doubles every hour or shows sequence (pattern) of numbers of bacteria in 1 hour intervals: 1 000, 2 000, 4 000, 8 000, 16 000, 32 000.
12	32 000. States that the numbers form a geometric series with common ratio $r = 2$ OR uses $S_n = ar^{n-1}$ for $r = 2$ OR uses an exponential equation in the general form of $y = A(a^k)$ with $A = 1000$, $a = 2$, and $K = 5$.
13	32 000. Uses an exponential equation involving e such as $y = 1000\,(e^{kt})$, $k = 0.6931$, $t = 5$.
19	Other correct responses.
Incorrect Response	
70	Answers other than 16 000 and 64 000. No work shown.
71	16 000 or 64 000. Exponential equation or pattern has been recognized correctly but there is a numerical error.
72	Responses other than 16 000 and 64 000 where a correct exponential has been used but there is a numerical or algebraic error. *Examples: $s_n = ar^{n-1}$* $y = A\,(a^k)$
73	Responses where the exponential function of the form $y = A(e^x)$ has been used but a numerical or algebraic error is made.
79	Other incorrect responses.
Nonresponse	
90	Crossed-out, illegible, or impossible to interpret.
99	BLANK

K14. A string is wound symmetrically around a circular rod. The string goes exactly 4 times around the rod. The circumference of the rod is 4 cm and its length is 12 cm.
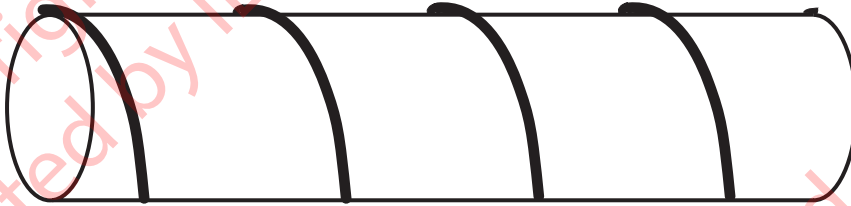
Find the length of the string. Show all your work.

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	next page	Geometry	Solving Problems	10%	752

K14. A string is wound symmetrically around a circular rod. The string goes exactly 4 times around the rod. The circumference of the rod is 4 cm and its length is 12 cm.

Find the length of the string. Show all your work.

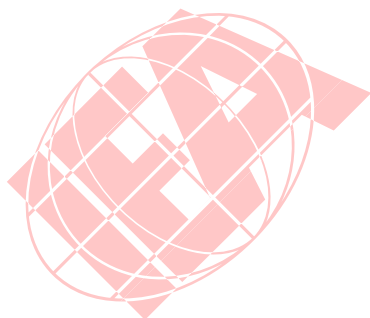| Code	Response
	Correct Response
20	Length of string = 20 cm. Method: • The surface of the rod is represented as a rectangle 4 cm by 12 cm. • Four parallel congruent segments are drawn in the rectangle indicating the position of the string. • Length of one segment is calculated using Pythagorean theorem $\sqrt{3^2 + 4^2}$ = 5. Total length of string = 4 x 5 cm = 20 cm.
21	Length of string = 20 cm. Method: • Half of surface of rod represented as rectangle 2 cm by 12 cm. • Eight congruent segments drawn in the rectangle indicating position of string. • Length of one segment calculated using Pythagorean theorem $\sqrt{2^2 + 1.5^2}$ = 2.5. Total length of string = 8 x 2.5 cm = 20 cm.
22	Length of string = 20 cm. Method used: • Situation represented either by rectangle 16 x 12 with string as its diagonal OR by right triangle with sides 16 and 12 and string as its hypotenuse. Pythagorean theorem used to calculate length of string $\sqrt{16^2 + 12^2}$ = 20 cm.
29	All other fully correct solutions.
	Partial Response
10	Length of string = 20 cm. No work shown.
11	Surface of rod represented by rectangle with correct dimensions and position of string correctly indicated, but numerical error in the calculation of the length of string.
19	All other partially correct solutions with <u>correct method</u> and minor error.

	Incorrect Response
70	Incorrect answer. No work shown.
71	Length of string = 16 cm. Argument: It is the same as 4 circles.
72	Length of string = 28 cm. Argument: "If the string were wound 4 times around the same place, its length would be 4 x 4. But since it "moves" along the rod which is 12 cm long, we must add these 12 cm to the length of the string."

73	Estimation methods:
	Length of 1 revolution estimated or stated but not calculated; then it is multiplied by 4.
	
	    Examples: 1 revolution is approx. 6 cm long, length of string is 4 x 6
	        = 24 cm.
	
	    1 revolution is (4 + 1.5) cm long, length of string is 4 x 5.5
	        = 22 cm.
	
	    Length of string must be greater than 16 cm (it would be 16
	     cm if it were 4 circles) and/or
	
	    Length of string must be less than 16 + 12 = 28 cm.
	
	    16 cm < L < 28 cm)

74	String is represented by a curve, e.g. parts of a circle or an ellipse.
79	All other incorrect attempts with some work shown.
	Nonresponse
90	Crossed-out, illegible, or impossible to interpret.
99	BLANK

K15. Determine all complex numbers z that satisfy the equation

$$z + 2\bar{z} = 3 + i$$

where $\bar{z}$ denotes the conjugate of z .

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	next page	Numbers, Equations and Functions	Routine Procedures	17%	696

K-15 Coding Guide

Code	Response
	Correct Response
20	z = 1 - i. No work shown
21	z = 1 - i. Method: Let z = a + bi. The given equation is then equivalent to 3a - ib = 3 + i OR 3(a-1) - i (b + 1) = 0. This equation correctly solved, finding a = 1, b = -1.
29	z = 1 - i, obtained by any other correct method.
	Partial Response
10	Equation for a,b is derived correctly, but either left unsolved or solution contains numerical or single algebraic error.
11	Due to numerical error an incorrect equation for a,b has been derived and solved, either correctly or incorrectly.
19	Other partially correct solutions with correct method but contains a numerical or single algebraic error.
	Incorrect Response
70	Incorrect answer. No work shown.
71	Attempts using b - ai as the conjugate of z, which leads to z = -3 - i.
72	Attempts using -z as the conjugate of z which leads to z = -1/3 + 5/3i.
73	Attempts using 1/z as the conjugate of z.
79	Other incorrect responses.
	Nonresponse
90	Crossed-out, illegible, or impossible to interpret
99	BLANK

K16. The ride with the cable car from station A to station B at the top of Mt. Glacier takes 16 minutes. The average speed of the cable car is 2 meters per second and it moves in a straight line forming a 25° angle with the horizontal.

Find the height of Mt. Glacier (measured from the level of station A) to the nearest meter. Show all your work.

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	next page	Numbers, Equations and Functions	Solving Problems	33%	631

K16. The ride with the cable car from station A to station B at the top of Mt. Glacier takes 16 minutes. The average speed of the cable car is 2 meters per second and it moves in a straight line forming a 25° angle with the horizontal.
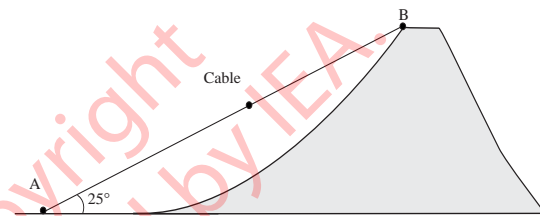

Find the height of Mt. Glacier (measured from the level of station A) to the nearest meter. Show all your work.

| Code	Response
	Correct Response
30	811 m or 0.811 km. Method: AB = 16 x 60 s ($2\text{m}/\text{s}$) = 1920m; height = 1920 sin 25° m = 8.114270625 m. Then rounds correctly to 811 m, or 0.811 km.
31	811 m or 0.811 km. Method: First AB calculated, then cos 25° used to determine AC and then the Pythagorean theorem used to find CB as $\sqrt{AB^2 - AC^2}$. **Note**: C denotes the point vertically beneath station B at the level of A. Height = $\sqrt{(1920)^2 - (1920Cos25°)^2}$ = 811.4270625 m. Then rounds correctly to 811 m, or 0.811 km.
39	Other complete correct responses.
	Partial Response
20	As code 30, but numerical result is not rounded.
21	1920 sin 25° as in code 30, but numerical value of expression is either not given or is calculated incorrectly.
22	As code 31, but numerical result is not rounded correctly.
23	Distance AB calculated incorrectly due to wrong method and/or numerical error in code 30 or code 31; the rest is correct.
24	As code 31, but value given is incorrect due to numerical error(s), other than in calculation of AB (code 21).
29	Other nearly complete solutions with a numerical error.

	Minimal Response
10	Distance AB found to be 1920 m, height calculated as 1920/sin 25 (leads to 4543 m) or 1920.cos 25 (leads to 1740 m) or 1920.tg 25 (leads to 895m). Numerical answer is given correctly or incorrectly or is not given at all.
11	Distance AB found to be 1920 m. Other work incorrect EXCEPT as stated in code 10 or impossible to interpret.
19	Other minimally correct solutions with not more than a total of two algebraic or trigonometric errors.
	Incorrect Response
70	Distance AB incorrectly calculated and wrong method(s) used to find height.
79	Other incorrect attempts.
	Nonresponse
90	Crossed-out, illegible, or impossible to interpret.
99	BLANK

K17. The graph of the function g passes through the point $(1,2)$. The slope of the tangent to the graph at any point (x, y) is given by $g'(x) = 6x - 12$. What is $g(x)$? Show all your work.
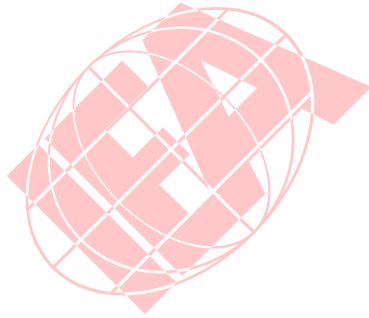
Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	next page	Calculus	Solving Problems	28%	642

K-17 Coding Guide

Code	Response
	Correct Response
30	$g(x) = 3x^2 - 12x + 11$. Method: First the function *g* is determined to be of the form $g(x) = 3x^2 - 12x + c$. Then c is found to be 11 by solving the equation $g(1) = 2$, i.e. $3 - 12 + c = 2$.
39	$g(x) = 3x^2 - 12x + 11$. Other correct method used.
	Partial Response
20	$g(x) = 3x^2 - 12x + 11$. No work shown.
21	Answer of the form $g(x) = 3x^2 - 12x + c$, where c is a numerical value other than 11. Method correct; the value of c given incorrectly due to error in solving the equation $g(1) = 2$. If c = 13, check to determine if 13 results from an error in solving $g(1) = 2$ [code 21] or from solving correctly the "reversed" equation, $g(2) = 1$ [code 22].
22	Method: Solves the "reversed" equation $g(2) = 1$ and determines the value of c to be 13. If an error is made in solving the reversed equation; c determined to be a number other than 13, use code 11.
23	Answers of the form $g(x) = 3x^2 - 12x + c$, where the value of c is NOT determined by solving either $g(1) = 2$ or $g(2) = 1$.
29	Other nearly complete solutions with a minor error.
	Minimal Response
10	Answer of the form $g(x) = 3x^2 - 12x + c$, where a numerical value of c is NOT found.
11	Method: Incorrectly solves "reversed" equation $g(2) = 1$, finds c to be a number other than 13.
19	Other minimally correct solutions
	Incorrect Response
70	$g(x) = 3x^2 - 12x$ and no work shown.
71	Answers of the form $g(x) = 6x^2 - ...$ or other integration error.
79	Other incorrect attempts.
	Nonresponse
90	Crossed-out, illegible, or impossible to interpret.
99	BLANK

K18. In the ΔABC the altitudes BN and CM intersect at point S. The measure of ∠MSB is 40° and the measure of ∠SBC is 20°. Write a PROOF of the following statement:

"ΔABC is isosceles."

Give geometric reasons for statements in your proof.

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	next page	Geometry	Justifying and Proving	34%	626

K-18 Coding Guide

K18. In the ΔABC the altitudes BN and CM intersect at point S. The measure of
∠MSB is 40° and the measure of ∠SBC is 20°. Write a PROOF of the
following statement:

"ΔABC is isosceles."

Give geometric reasons for statements in your proof.
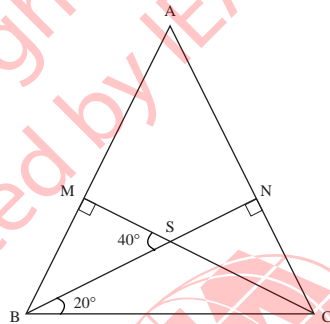
Note: To be considered correct, all responses must include mention of all geometric facts used, all
calculations made, and a conclusion

Code	Response
Correct Response	
20	Correct proof. Proves that ∠B = ∠ C using the following facts: • the sum of angles in any triangle is 180°. • if two angles of a triangle are equal, the triangle is isosceles. and possibly also uses: • vertically opposite angles are equal. • supplementary angles add to 180°. The concept of congruence is not used.
21	As code 20 but somewhere in the proof uses the fact that some triangles: e.g. triangles BCM and CBN, OR triangles BMS and CNS, are congruent.
29	All other fully correct and complete proofs.
Partial Response	
10	As in codes 20-21 shows ∠B and ∠C are equal giving steps in logical order, but omits one step or one reason or gives one incorrect reason.
11	As in codes 20-21 shows ∠B and ∠C are equal, states correct geometric facts but not in a logically correct order.
19	Other responses with minor errors.
Incorrect Response	
70	Shows measures of angles correctly on figure but no geometric facts mentioned or argumentation given.
71	Incorrect argumentation and/or includes more than one incorrect geometric fact, step, or reason.
72	"Proof" is circular; makes use of statements which are equivalent to what is to be proven.
79	Other incorrect responses.
Nonresponse	
90	Crossed-out, illegible, or impossible to interpret.
99	BLANK

L1. What are all values of x for which the inequality $5x + \dfrac{5}{3} \leq -2x - \dfrac{2}{3}$ is true?

A. $x \leq -\dfrac{7}{9}$

B. $x \leq -\dfrac{1}{3}$

C. $x \geq 0$

D. $x \geq \dfrac{7}{3}$

E. $x \geq \dfrac{9}{3}$

L-1

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	B	Numbers, Equations and Functions	Routine Procedures	73%	444

L2. Given $\log_b 2 = \dfrac{1}{3}$, $\log_b 32$ is equal to

A. 2

B. 5

C. $-\dfrac{3}{5}$

D. $\dfrac{5}{3}$

E. $\dfrac{3}{\log_2 32}$

L-2

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	D	Numbers, Equations and Functions	Routine Procedures	63%	505

L3. A radio-active element decomposes according to the formula,

$$y = y_o e^{-kt}$$

where y is the mass of the element remaining after t days and y_o is the value of y for $t = 0$.

Find the value of the constant k for an element whose half-life (i.e. time to decompose half of the material) is 4 days.

A. $\frac{1}{4}\log_e 2$

B. $\log_e \frac{1}{2}$
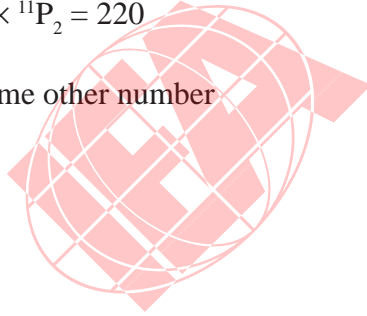
C. $\log_2 e$

D. $(\log_e 2)^{\frac{1}{4}}$

E. $2e^4$

L-3

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	A	Numbers, Equations and Functions	Complex Procedures	44%	610

L4. An examination consists of 13 questions. A student must answer only one of the first two questions and only nine of the remaining ones. How many choices of questions does the student have?

A. $^{13}C_{10} = 286$

B. $^{11}C_8 = 165$

C. $2 \times {}^{11}C_9 = 110$

D. $2 \times {}^{11}P_2 = 220$

E. some other number

L-4

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	C	Numbers, Equations and Functions	Solving Problems	48%	582

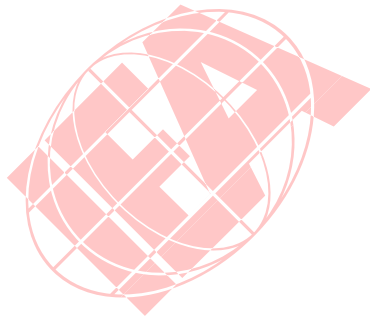L5.   The sum of the infinite geometric series $1 - \dfrac{1}{2} + \dfrac{1}{4} - \dfrac{1}{8} + \ldots$ is

A. $\dfrac{5}{8}$

B. $\dfrac{2}{3}$

C. $\dfrac{3}{5}$

D. $\dfrac{3}{2}$

E. ∞

L-5

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	B	Calculus	Routine Procedures	45%	597

L6. The velocity v of a body moving in a straight line t seconds after starting from rest is $v = 4t^3 - 12t^2$ meters per second.

How many seconds after starting does its acceleration become zero?

A.   1

B.   2

C.   3

D.   4
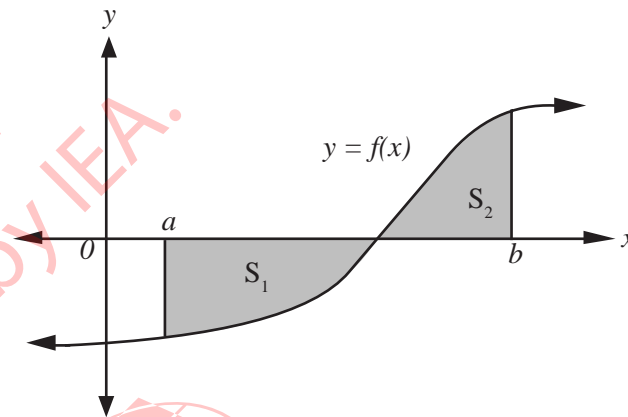
E.   6

L-6

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	B	Calculus	Routine Procedures	33%	669

L7.

This figure shows the graph of $y = f(x)$.
S_1 is the area enclosed by the x-axis, $x = a$ and $y = f(x)$;
S_2 is the area enclosed by the x-axis, $x = b$ and $y = f(x)$;
where $a < b$ and $0 < S_2 < S_1$.

The value of $\int_a^b f(x)\,dx$ is

A. $S_1 + S_2$

B. $S_1 - S_2$

C. $S_2 - S_1$

D. $|S_1 - S_2|$
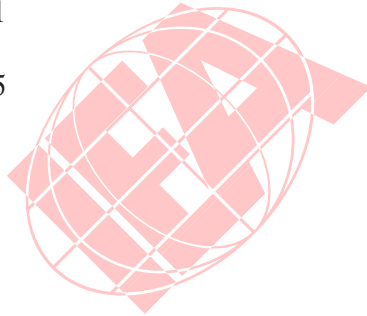
E. $\dfrac{1}{2}(S_1 + S_2)$

L-7

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	C	Calculus	Solving Problems	35%	658

L8. The rectangular coordinates of three points in a plane are $Q(-3, -1)$, $R(-2, 3)$, and $S(1, -3)$. A fourth point T is chosen so that $\overrightarrow{ST} = 2\overrightarrow{QR}$.

The y-coordinate of T is .

A.   −11

B.   −7

C.   −1

D.   1

E.   5

L-8

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	E	Geometry	Routine Procedures	50%	576

L9.
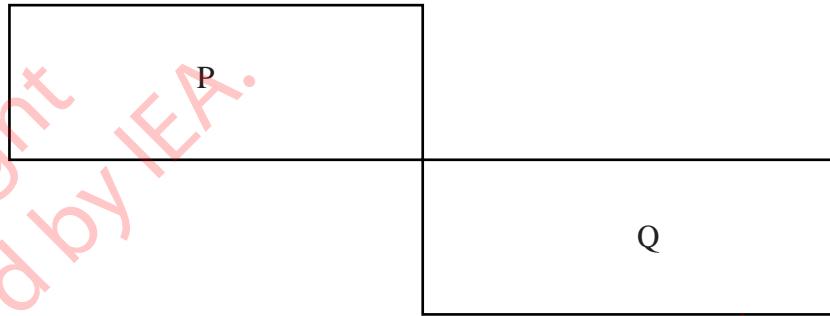
The rectangle labeled Q CANNOT be obtained from the rectangle labeled P by means of a

A.   reflection (about an axis in the plane of the page)

B.   rotation (in the plane of the page)

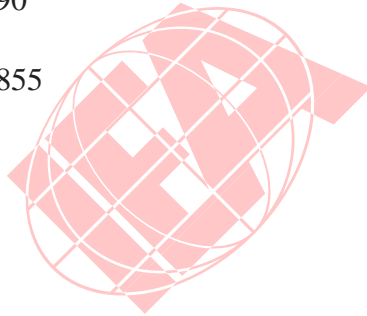C.   translation

D.   translation followed by a reflection

L-9

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	A	Geometry	Knowing	56%	546

L10. A warning system installation consists of two independent alarms having probabilities of operating in an emergency of 0.95 and 0.90 respectively. Find the probability that at least one alarm operates in an emergency.

A.   0.995

B.   0.975

C.   0.95

D.   0.90

E.   0.855

L-10

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	A	Probability & Statistics	Solving Problems	29%	691

L11. The Smith sisters made these statements. If Vera told the truth, who else must have told the truth?
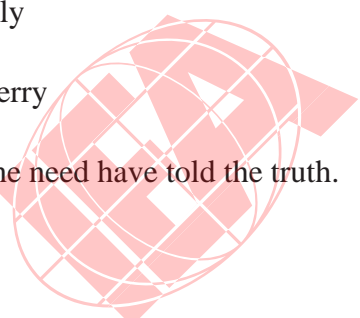
Lucy: "If the rug is in the car, then it is not in the garage."
Sally: "If the rug is not in the car, then it is in the garage."
Vera: "If the rug is in the garage, then it is in the car."
Cherry: "If the rug is not in the car, then it is not in the garage."

A.   Lucy

B.   Sally

C.   Cherry

D.   none need have told the truth.
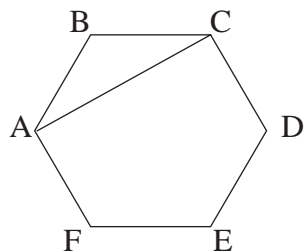
L-11

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	C	Validation and Structure	Routine Procedures	76%	425

L12. Each side of the regular hexagon ABCDEF is 10 cm long. What is the length of the diagonal AC?

A. $10\sqrt{3}$ cm

B.   20 cm
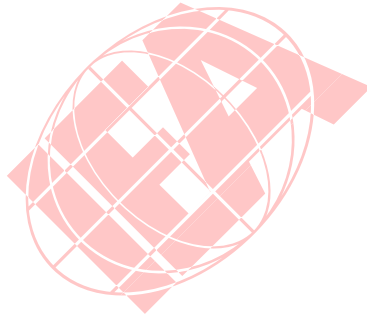
C. $5\sqrt{3}$ cm

D.   10 cm

E. $20\sqrt{3}$ cm

L-12

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	A	Geometry	Solving Problems	66%	486

L13. Two vectors $\vec{a}$ and $\vec{b}$ ($\vec{a}$, $\vec{b} \neq 0$) are related by: $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$.

What is the measure of the angle between $\vec{a}$ and $\vec{b}$?

L-13

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	next page	Geometry	Complex Procedures	29%	699

L-13 Coding Guide

L13. Two vectors $\vec{a}$ and $\vec{b}$ $(\vec{a}, \vec{b} \neq \vec{0})$ are related by: $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$.
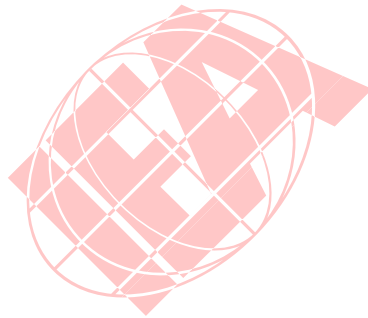
What is the measure of the angle between $\vec{a}$ and $\vec{b}$?

Code	Response
	Correct Response
10	90º OR ± π/2 OR "Given vectors are perpendicular." No work shown.
11	90º OR ± π/2 OR "Given vectors are perpendicular." And results derived <u>correctly</u> from equality given and from definition of |a|, that is, |a+b| = |a-b| ⇒ |a+b|^2= |a-b|^2 ⇒ (a+b) (a+b) ⇒ (a-b) (a-b) ⇒ a^2+ 2ab + b^2 $=$ a^2 - 2ab+b^2 ⇒ 2ab = -2ab ⇒ 4ab = 0 ⇒ vectors a,b are perpendicular. **Note:** If error(s) in derivation, code 19.
12	90º OR ± π/2 OR "Given vectors are perpendicular." Situation is represented correctly geometrically, that is, vectors a+b and a-b represented as diagonals of a parallelogram. Conclusion stated that the given equality holds only if vectors a,b are perpendicular to each other. **Note:** If error(s) in method, code 19.
13	90º OR ± π/2 OR "Given vectors are perpendicular." Only presents a drawing; no explanation in words.
19	90º OR ±π/2 OR "Given vectors are perpendicular." Some work shown. Method may be complete and correct, or incomplete and correct, or incorrect.
	Incorrect Response
70	0º or 180º with or without work shown.
71	Method as in code 11, but no statement of measurement.
72	Method as in code 12, but no statement of measurement.
73	Unsuccessful atttempts to use the formula: ab cos α = |a| · |b| .
79	All other incorrect responses.
	Nonresponse
90	Crossed-out, illegible, or impossible to interpret.
99	BLANK

L14. One thousand people selected at random were questioned about smoking and drinking. The results of this survey are summarized in the table below. Calculate the probability that a randomly selected respondent drinks and smokes.

	Smokers	Non-smokers
Drinkers	320	530
Non-drinkers	20	130

L-14

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	next page	Probability & Statistics	Solving Problems	51%	570

L-14 Coding Guide

L14. One thousand people selected at random were questioned about smoking and drinking. The results of this survey are summarized in the table below. Calculate the probability that a randomly selected respondent drinks and smokes.

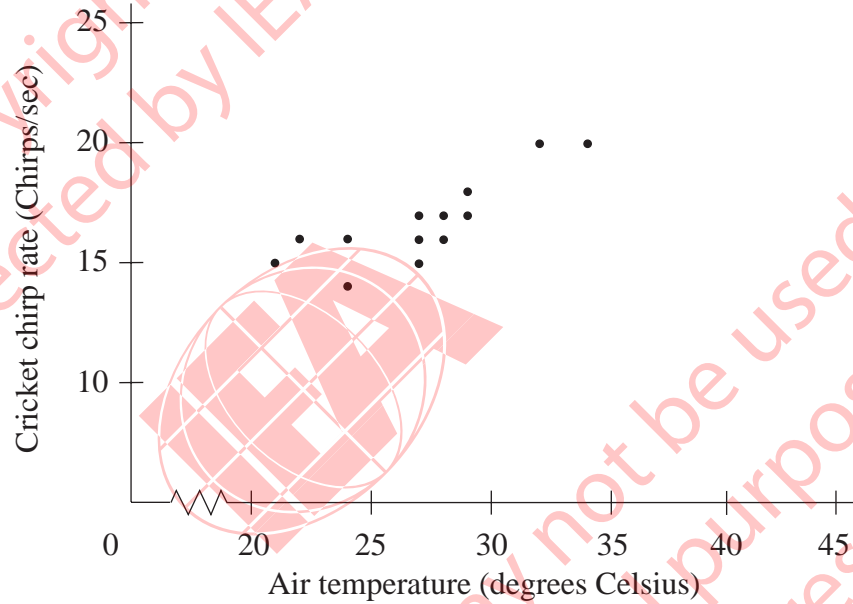|	Smokers	Non-smokers
Drinkers	320	530
Non-drinkers	20	130

Code	Response
	Correct Response
10	0.32 or any numerical equivalent such as $\dfrac{320}{1000}$ or 32%. If probability is correct, disregard rounding errors.
	Incorrect Responses
70	0.032 or similar answers involving place value error.
71	320
72	850/1000 or 340/1000
73	1/320 or 1000/320
79	Other incorrect responses.
	Nonresponse
90	Crossed-out, illegible, or impossible to interpret.
99	BLANK

L15. Scientists have observed that crickets move their wings faster in warm temperatures than in cold temperatures. By noting the pitch of cricket chirps, it is possible to estimate the air temperature. Below is a graph showing 13 observations of cricket chirps per second and the associated air temperature.

a) On the graph, draw in an estimated line of best fit for these data.

b) Using your line, estimate the air temperature when cricket chirps of 22 per second are heard.

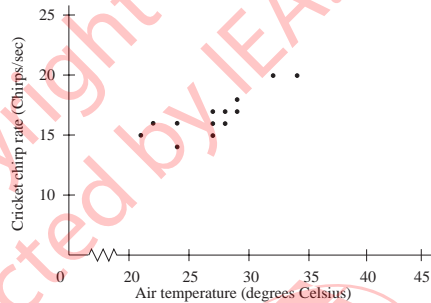Estimated air temperature:_____

L-15a

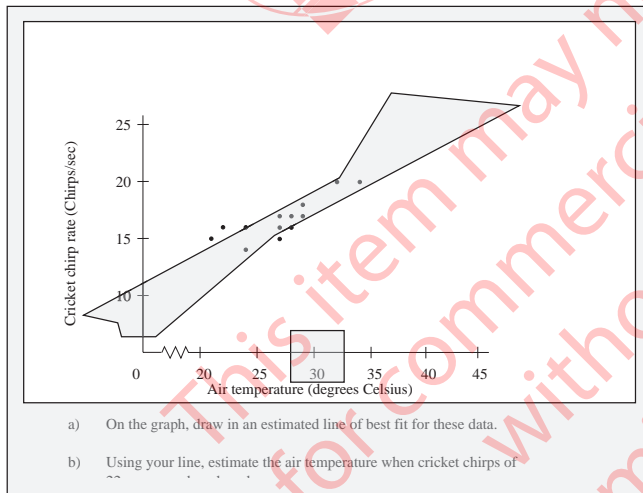|	Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Part a	Advanced Mathematics	next page	Probability & Statistics	Complex Procedures	52%	570

L-15a Coding Guide

L15. Scientists have observed that crickets move their wings faster in warm temperatures than in cold temperatures. By noting the pitch of cricket chirps, it is possible to estimate the air temperature. Below is a graph showing 13 observations of cricket chirps per second and the associated air temperature.
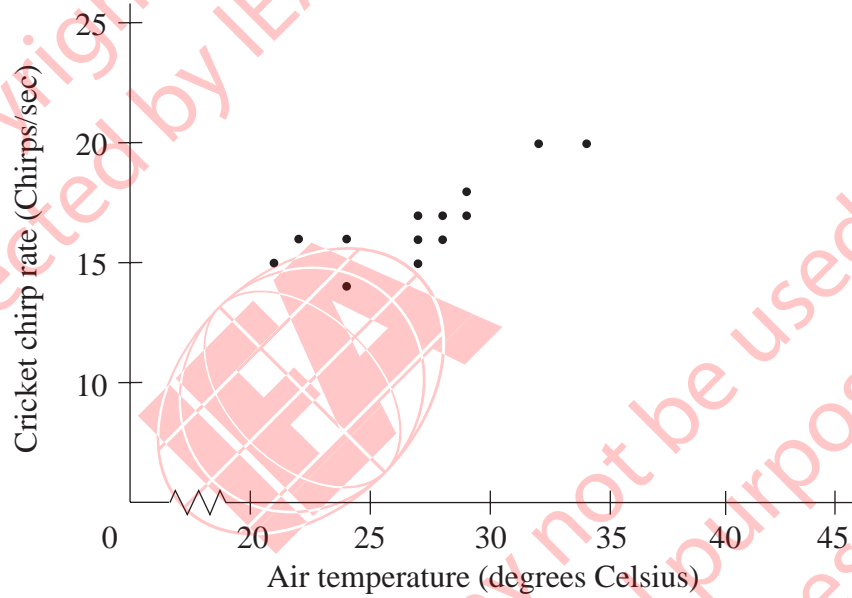

a) On the graph, draw in an estimated line of best fit for these data.

b) Using your line, estimate the air temperature when cricket chirps of 22 per second are heard.

Estimated air temperature:_____

Example of template showing allowable range

a) On the graph, draw in an estimated line of best fit for these data.

b) Using your line, estimate the air temperature when cricket chirps of

A: Codes for Line of Best Fit

Code	Response
Correct Response	
10	When template is lined up with the origin, the straight line of best fit should appear in the cut out section of the template for all air temperatures from 20 to 45 degrees Celsius.
Incorrect Response	
70	The straight line of best fit does not appear in the "cut out" section of the template for all air temperatures from 20 to 45 degrees Celsius.
71	The graph is NOT a straight line, e.g., it is a curve or zig-zag line.
Nonresponse	
90	Crossed-out, illegible, or impossible to interpret.
99	BLANK

L15. Scientists have observed that crickets move their wings faster in warm temperatures than in cold temperatures. By noting the pitch of cricket chirps, it is possible to estimate the air temperature. Below is a graph showing 13 observations of cricket chirps per second and the associated air temperature.

a) On the graph, draw in an estimated line of best fit for these data.

b) Using your line, estimate the air temperature when cricket chirps of 22 per second are heard.

Estimated air temperature:_____

L-15b

Part b

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	next page	Probability & Statistics	Complex Procedures	64%	498

L-15b Coding Guide

L15. Scientists have observed that crickets move their wings faster in warm temperatures than in cold temperatures. By noting the pitch of cricket chirps, it is possible to estimate the air temperature. Below is a graph showing 13 observations of cricket chirps per second and the associated air temperature.
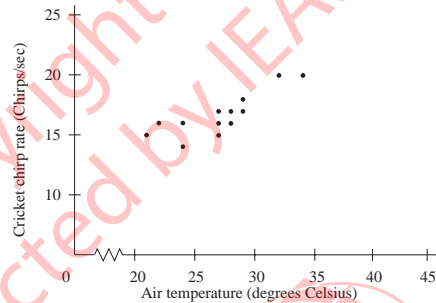
a) On the graph, draw in an estimated line of best fit for these data.

b) Using your line, estimate the air temperature when cricket chirps of 22 per second are heard.

Estimated air temperature:_____

B: Codes for Estimate Air Temperature

Code	Response
	Correct Response
10	For code 10 in part A, the answer should appear to be a correct projection from the student's straight line of best fit and must be in the range of 34 to 42 degrees Celsius inclusive. Student's answer should be within ± 2 degrees Celsius of the correct estimate based on the student's line of best fit.
11	For code 70 in part A, the answer is not necessarily in the range of 34 to 42 degrees Celsius but should appear to be a correct projection from the student's straight line of best fit within ± 2 degrees Celsius.
	Incorrect Response
70	The answer is NOT a reasonable projection from the student's straight line of best fit.
71	The answer (estimate) is based on a curved or zig-zag line.
	Nonresponse
90	Crossed-out, illegible, or impossible to interpret.
99	BLANK

L16. Find all real values of x which satisfy the following equation:

$$\sqrt{x} - \frac{2}{\sqrt{x}} = 1$$

Show all your work.

L-16

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	next page	Numbers, Equations and Functions	Solving Problems	24%	664

L16. Find all real values of x which satisfy the following equation:

$$\sqrt{x} - \frac{2}{\sqrt{x}} = 1$$

Show all your work.

| Code	Response
Correct Response	
30	$x = 4$. Method: 1. Original equation transformed to quadratic equation, $x^2 - 5x + 4 = 0$. 2. Two roots, x=4 and x=1 found and <u>checked in original equation</u>. 3. x=1 is rejected as a solution of the original equation; x=4 is accepted. **Note:** Since the original equation is squared, it is necessary to check both roots in the <u>original equation</u>.
31	$x = 4$. Method: 1. Substitution (e.g., $\sqrt{x} = a$) used and the original equation transformed, without squaring, into the quadratic equation $a^2 - a - 2 = 0$. 2. Two roots found, a = 2 and a = -1. 3. a = -1 rejected since a = -1 $\neq \sqrt{x}$, $\sqrt{x2}$. 4. By substitution reversed.a = 2 implies $\sqrt{x} = 2$, thus x = 4. Checking in original equation is not necessary.
32	$x = 4$. Method: 1. Graphs $y = \sqrt{x} - \frac{2}{\sqrt{x}} - 1$ for x > 0 correctly. 2. x-coordinate of y-intercept is found to be 4. 3. Justifies that graph is increasing and thus x = 4 is an unique solution. 4. x = 4 is <u>checked in original equation</u>.
39	Other completely correct solutions.
Partial Response	
20	Uses code 30 to find x = 4 and x = 1 and states both are roots.
21	Uses code 31 to find a = 2 and a = -1 and then either goes no further or makes an incorrect statement such as 4 and 1 or 2 and 1 are roots of the original equation.
22	Uses code 32 showing graph, states x = 4 is a root and x = 4 is checked in the original equation.
29	Other solutions with correct overall method but with minor error(s).

	Minimal Response	
10	x = 4. No work shown or some work shown, such as checking x = 4 in original equation but no argument given for why there are no other roots.	
11	Solution as in codes 30 or 31: Original equation is transformed correctly into a quadratic equation, by any method, but quadratic equation either is not solved or incorrectly solved.	
12	Solution as in code 22 (graphical) except that except that x = 4 is <u>NOT</u> checked in original equation.	
19	Other minimally correct or incomplete solutions such as a simplification of the equation to $x - 2 = \sqrt{x}$.	
	Incorrect Response	
70	Solution as in codes 30 or 31 except original equation is transformed into an incorrect quadratic equation or to a non-quadratic equation.	
79	Other incorrect responses.	
	Nonresponse	
90	Crossed-out, illegible, or impossible to interpret.	
99	BLANK	

L17. For what real value of k will the equation below describe a circle with radius 3?

$$x^2 + y^2 + 2x - 4y + k = 0$$

Show all your work.
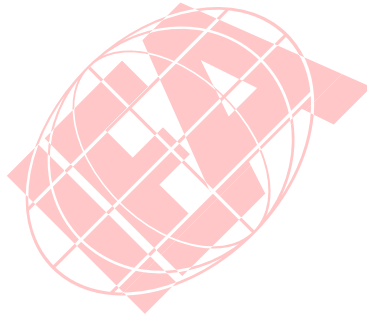
L-17

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	next page	Geometry	Communicating	20%	697

L-17 Coding Guide

Code	Response
Correct Response	
20	K = -4. Method used: • Because of the quadratic and linear terms, the equation must be of the general form $(x+1)^2 + (y-2)^2$. From that we get the equation • $(x+1)^2 + (y-2)^2 -5 + k = 0$ • $(x+1)^2 +(y-2)^2 = 5 - k$ • If the radius is 3, right hand side must equal 9 (= r^2) • Hence 5 - k = 9 and k = -4 is the only solution.
21	k = -4. Method used: • All circles with radius 3 have same general form: $x^2 + y^2 - 2ax - 2by + a^2 + b^2 - 9 = 0$. • From that: 2 = -2a; -4 = -2b; k = $a^2 + b^2$ -9; • Hence a = -1, b = 2, k = -4.
29	Any other fully correct solution.
Partial Response	
10	k = -4. No work shown.
11	Method as in code 20 but with numerical error(s) only.
12	Method as in code 21 but with numerical error(s) only.
19	All other partially correct solutions.
Incorrect Response	
70	Incorrect answer. No work shown.
71	k = -12 with or without work shown. [This answer can be obtained by the misconception that point (3,3) is on the circle, hence x = 3 and y = 3 are put into the equation.]
72	k = 3 OR k = 9 OR k = -9 with or without work shown. [This answer can be obtained by the miscoknception that the parameter k represents the radius or square of radius of the circle.]
73	k = 8 or k = 2 or k = 14 with or without work shown. [This answer can be obtained by the misconception that -5 + k = 3 OR 5 - k = 3 OR -5 + k = 9.]
79	All other incorrect responses with some work shown.
Nonresponse	
90	Crossed-out, illegible, or impossible to interpret.
99	BLANK

L18. Two circles with centres A and B as shown below have radii of 7 cm and 10 cm respectively. If the length of the common chord PQ is 8 cm, what is the length of AB? Show all your work.

L-18

Subject	Item Key	Content Category	Performance Expectation	International Average Percent of Students Responding Correctly	International Difficulty Index
Advanced Mathematics	next page	Geometry	Solving Problems	50%	573

L-18 Coding Guide

L18. Two circles with centres A and B as shown below have radii of 7 cm and
10 cm respectively. If the length of the common chord PQ is 8 cm, what is the
length of AB? Show all your work.
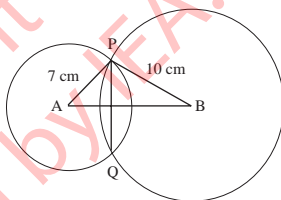
Note: 1. Since the expected precision is not indicated, every result obtained by a correct method
and reasonable and correct rounding should be accepted.
2. If student gives at some stage the correct answer but continues and later makes a
numerical (not conceptual) error, ignore this error.

Code	Response
Correct Response	
20	14.9 or $\sqrt{84} + \sqrt{33}$. Method: Pythagorean theorem applied in triangles APS and BPS (S is the midpoint of PQ).
21	14.9. Method: Trigonometry functions (ratios) used correctly to determine lengths of sides of triangles OR size of angles of triangle that results in determining the length of AB. **Note:** Most frequently used trigonometric functions are sine and cosine.
29	Other complete and correct solutions.
Partial Response	
10	Method as in 20 but solution contains a (minor) error in method, or numerical or rounding error.
11	Method as in 21 but solution contains a (minor) error in method, or numerical or rounding error.
12	$\sqrt{84} + \sqrt{33}$. No work shown.
19	Other partially correct solutions, with minor error.
Incorrect Response	
70	Method: Pythagorean theorem applied to ΔAPB which is not a right triangle.
71	Incorrect use of the Pythagorean theorem in a right triangle.
72	Figure in booklet has been considered accurate and lengths of segments and/or measures of angles have been determined from the diagram.
79	All other incorrect responses.
Nonresponse	
90	Crossed-out, illegible, or impossible to interpret
99	BLANK

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